ST227 - Exercise 2

Question 1

The lifetime of a mechanical system is modelled by the following mortality intensity:

$$\mu(t) = \lambda, \quad t \geq 0, \quad \lambda = 10^{-1}\,.$$

a. Define in R the survival probability function $(t, x) \to {}_tp_x$ and calculate the probability of surviving the next 5 years for a 15-year-old system.

General wear and tear in mechanical systems mean the memoryless property of constant mortality is a questionable one. We overcome that by introducing a time-dependent term:

$$\tilde{\mu}(t) = \lambda + \gamma \log(\log(e + t)), \quad t \geq 0, \quad \lambda = 10^{-1}, \gamma = 1.5\,.$$

Consider a 15-year-old machine, of which the remaining lifetime T_{15} follows the mortality function $\tilde{\mu}(t)$.

b. Define in R the density function for T_{15}. This definition may involve a numerical integral.

c. Calculate the expected remaining lifetime for this individual.

d. Define in R the cumulative distribution function of T_{15}. This definition may involve a numerical integral. Discuss how you would find the 95-th percentile of T_{15}.

Question 2

Consider the constant morstality function

$$\mu(t) = 0.05\,.$$

a. Calculate the probability that individuals aged 20, 40 and 80 will survive the next 20 years.

A class of commonly considered mortality is called Gompertz-Makeham's (or simply Makeham's) mortality, which has the form:

$$A + Bc^x\,.$$

b. Using Makeham's mortality with $A = 5 \times 10^{-4}$, $B = 7.5858 \times 10^{-5}$ and $c = 1.09144$, calculate the probability that individuals aged $20, 40$ and 80 will survive the next 20 years.

c. Denote by K_x the remaining curtate lifetime of an individual aged x . Then

$$K_x = \lfloor T_x \rfloor\,,$$

where the mapping $t \to \lfloor t \rfloor$ is the floor function. Calculate the curtate lifetime of a 20-year old individual with Makeham's mortality function.

d. It can be shown that

$$\mathrm{E}(K_x) = \sum_{i=1}^{\infty} {}_np_x\,.$$

Utilise this formula to calculate the expected curtate lifetime. Approximate the infinite sum by the first 100 terms.

Question 3

Consider the following mortality intensity function

$$\mu(t) = \frac{\alpha\gamma t^{\gamma-1}}{1+\alpha t^{\gamma}}, \quad \alpha = 3.757 \times 10^{-2.2}, \quad \gamma = 1.4243\,.$$

a. Define in R the survival probability function $(t, x) \to {}_tp_x$ and calculate the probability of surviving the next 3 years for a 20-year-old individual.

b. Define in R the density function for T_{20}.

c. Calculate the expected remaining lifetime for a 20-year-old individual.

d. Explain how you would find the median of this distribution.

Question 4

The time (in days) to failure, denoted T_x, of a mechanical system aged x can can be mod-lled by the following mortality (also known as the hazard function in non-life contexts)

$$\mu(t) = \lambda, \quad t \geq 0, \lambda = 0.05\,,$$

a. Define in R the survival probability function $(t, x) \to {}_tp_x$ and calculate the probability that a new system will operate without failures for 10 days.

b. Define in R the density function for T_0.

c. Calculate the expected number of days between until failure of a new system and that of a system that has been continuously running for 20 days. Comment on your finding.

d. Consider a modified version of the hazard function

$$\mu(t) = \lambda + 0.2\log(t+1)\,.$$

Compute $\mathrm{E}(T_0)$ and $\mathrm{E}(T_{20})$ again. Comment on their values and interpret the impact of the extra term in the modified hazard function.